Wide-Band Analog Function Multiplier

By

JOSEPH A. MILLER,
Minneapolis-Honeywell
Minneapolis, Minn.

AARON S. SOLTES
Air Force
Cambridge Research Center
Cambridge, Mass.

and

RONALD E. SCOTT
Research Laboratory of Electronics
MIT
Cambridge, Mass.

Beam-deflection tubes perform nonlinear squaring operations that are the basis of this analog multiplication method. Speed and accuracy are high. Performance is primarily limited by associated circuitry rather than the tubes

DEVELOPED on the quarter-square principle, a simple analog function multiplier takes advantage of the characteristics of recently developed beam-deflection square-law tubes, such as type QK-329. These can provide full parabolic square-law action to an accuracy better than 1 percent of full scale, over a frequency range from d-c to the vhf region.

This particular multiplier was built to explore the possibility of using these square-law tubes for this application. Commercially available plug-in amplifiers were employed in the associated circuits. Results obtained showed that performance of this relatively crude model was, on the whole, limited by the associated circuitry rather than the square-law tubes. Nevertheless a combination of accuracy and speed of response had been achieved that exceeded any other known method of analog multiplication.

A quarter-square multiplier is instrumented around the identity

$$xy = \tfrac{1}{4}(x + y)^2 - (x - y)^2 \qquad (1)$$

The left-hand term is the desired product and requires the performance on the right-hand side of the operations of addition, subtraction, multiplication by a constant, and squaring. All operations but squaring are linear and are readily accomplished using conventional techniques. The particular method selected for achieving the two nonlinear squaring functions, however, presents a design problem and is principally responsible for the characteristics that distinguish one quarter-square multiplier from another.

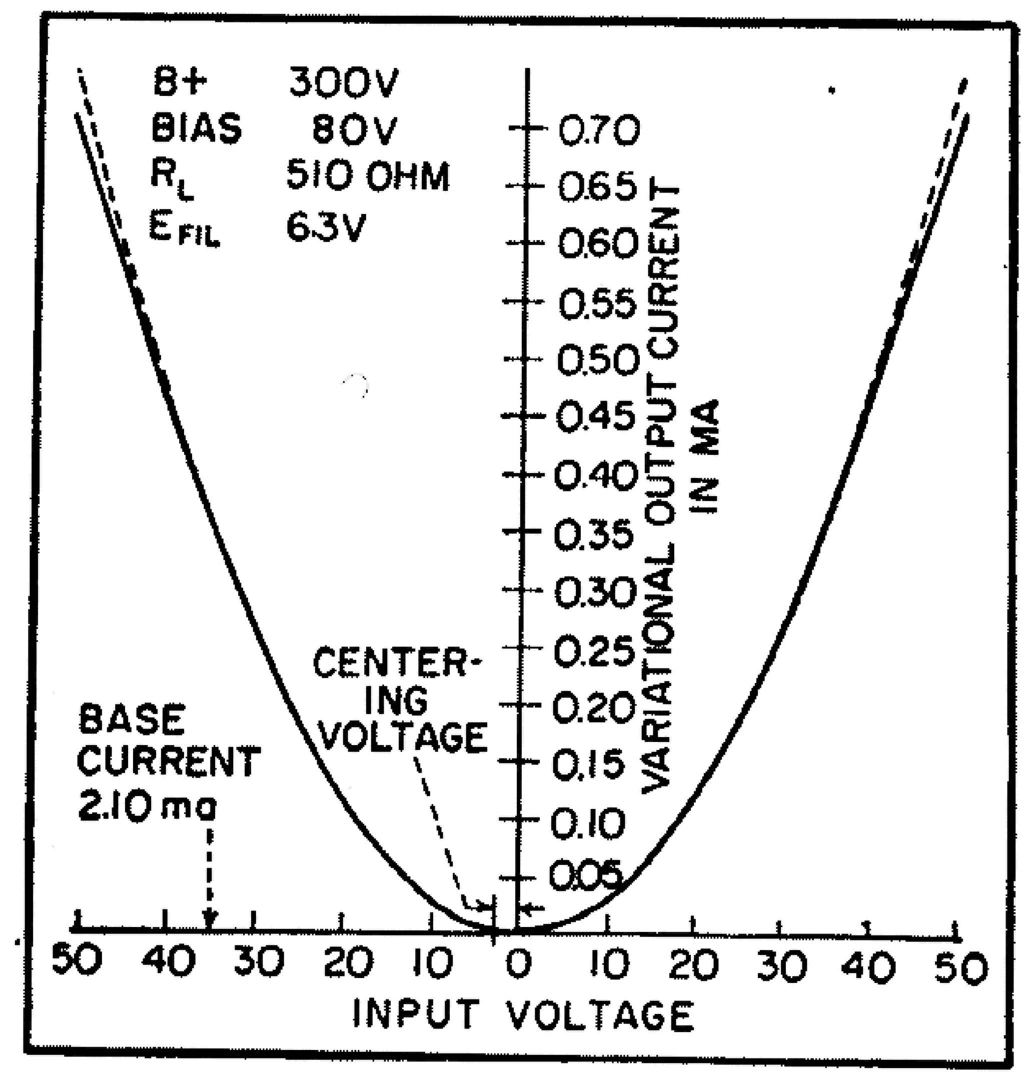


FIG. 1—Comparison of curved portion of static characteristic of square-law tube with parabola

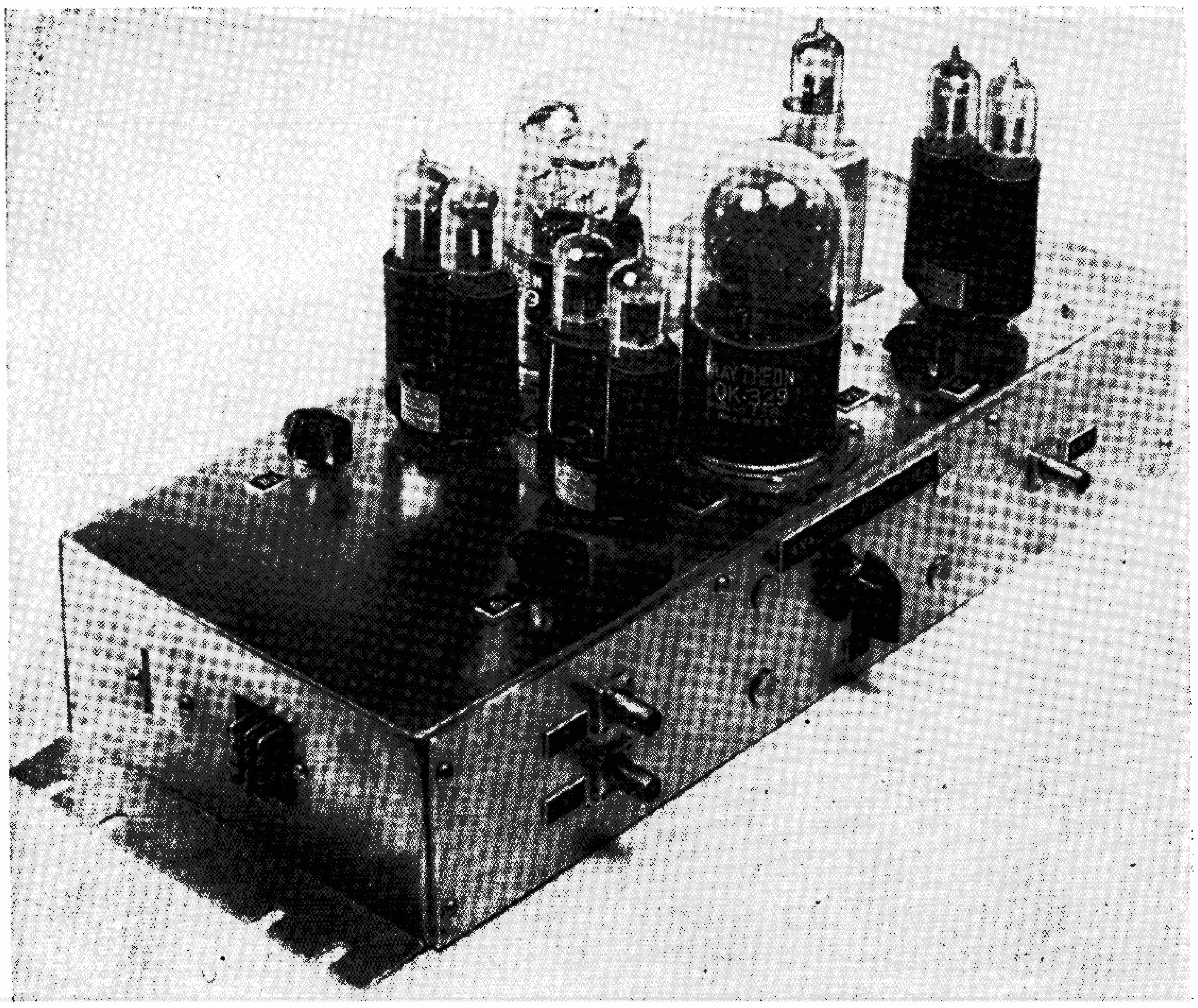
The beam-deflection square-law tubes used as the starting point in the present design are capable of providing accurate reproducible full-parabolic transfer characteristics in a noncritical manner. Over their range of operation, speed of response and accuracy capabilities are essentially independent of each other. That is, the same accuracy obtainable under d-c conditions is achievable at the upper end of its frequency range.

Square-Law Circuit Elements

The principle employed to produce a square-law characteristic in these tubes is that of deflecting a flat sheet of electrons across a target electrode containing parabolic apertures.

The discussed tube is essentially a copy, with only minor modifications of the simplest of the QK-256 series of experimental beam-deflection square-law tubes.[1] More precise methods of measuring and plotting the nonlinear static characteristics have indicated that the accuracies achievable are better than was previously stated.

Figure 1 is a plot of a typical



Complete analog multiplier employs amplifiers that provide a single-ended low-impedance output at an open-loop d-c gain of over 10,000. Present frequency response is limited by amplifier bandwidths

Type QK-329 beam-deflection tube provides full parabolic square-law action with 1 percent accuracy at full scale

static characteristic made on an automatic precision (0.1 percent) plotting board. Within an input range about the origin of approximately ±35 volts the error is too small to measure by such means and remains less than 1 percent up to ±40 volts. Within these limits the static characteristic may be idealized to a close approximation as a parabola

$$i_{out} = i_0 + k(e_0 + e_{in})^2 \text{ amp} \qquad (2)$$

with its vertex displaced from the origin by amounts e_0 and i_0.

Current

Scale factor k is expressed in mhos per volt and is essentially a constant for a given tube over a wide range of variations in cathode to anode voltage, when operated with its average deflection-plate potential, E_{bias}, maintained at a fixed fraction of E_{B+}. The voltage required to center the parabola on the vertical axis, $-e_0$, is generally small. Its magnitude may differ from tube to tube, but remains constant with time for a given tube and is not sensitive to changes in operating potentials. Self-centering schemes are, therefore unnecessary to hold e_0 constant.

The current, i_0, is a function of total current and subject to change when any of the operating parameters that affect total current are varied, such as heater voltage, E_{bias} or E_{B+}. Normal precautions appropriate to d-c amplifier design are therefore advisable to keep i_0 stable.

As with most beam-deflection devices, best operation of the QK-329 is obtained with a balanced input. The input conductance between deflection plates is small enough to ignore under most conditions. Where precise operation at d-c is required, account should be taken of the possible presence of diode currents of about 10 microamperes between the cathode and the positively biased deflection plates. This current is not an inherent property of the tube type, but rather a consequence of the fact that its effect had not been noticed in earlier applications.

The Multiplier

Equation 1 can be instrumented in a variety of ways. The block diagram of Fig. 2 illustrates the arrangement of functional components employed. No effort was made to obtain any particular overall multiplier scale factor. Three identical plug-in operational amplifier units are used in standard feedback computer configurations of unity gain to perform both the inverting and subtracting functions. These amplifiers provide a high-impedance differential pair of inputs and a single-ended low-impedance output at an open-loop d-c gain of over 10,000. Both amplifier inputs handle signals in the subtractor stage, whereas one of the inputs in each of the two inverter stages is only used for zeroing purposes.

The output or product of a multiplier with identical inputs may contain frequency components up to twice as high as those present in either input. Consequently, the subtractor must be capable of operating up to a maximum frequency of double that which must be handled by the inverters.

In order to further extend the frequency range of the subtractor it was found necessary to reduce the impedance level of its associated ex-

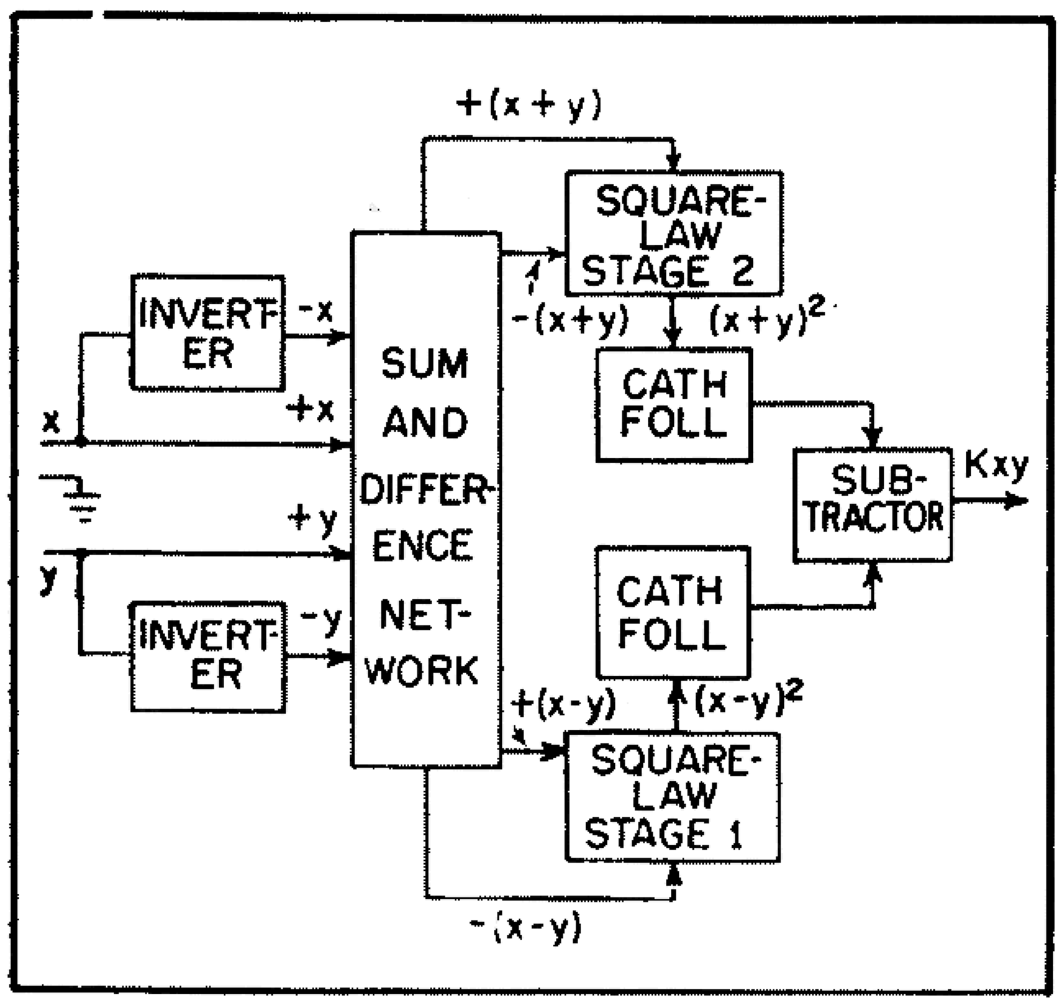


FIG. 2—Multiplier system using identical amplifier units for inverting and subtracting

ternal computing resistance below that used in the inverters.

Ground potential is used as an absolute zero reference for the multiplier input and output signals. The plug-in amplifier circuits when zeroed do not introduce any shift in reference level between their inputs and outputs. However, the input and output of a square-law stage normally operate at different potentials. The deflection plates of the beam-deflection tube rather than its output, operate near ground potential. This avoids complicating the driving circuits to the square-law stages, that are already burdened with provisions for forming balanced sum-and-difference signals.

The potential differences between square-law stage outputs and subtractor inputs are eliminated through the use of conventional voltage-divider step-down arrangements. To prevent excessive signal attenuation, cathode followers are inserted between the high-impedance voltage-divider taps and the lower impedance inputs to the subtractor.

Push-pull sum-and-difference signals for the square-law stage inputs, $\pm(x + y)$ and $\pm(x - y)$, are formed with respect to ground potential in a symmetrical passive summing network. To do this, the network is supplied with balanced versions of the multiplier input signals, $\pm x$ and $\pm y$. Signals $+x$ and $+y$ are derived directly from the input terminals of the multiplier, while the inverters provide their negative counterparts. A schematic diagram of the complete wide-band analog function multiplier appears in Fig. 3.

Instantaneous output accuracy within ±0.5 percent of maximum product was consistently achieved within the input operating ranges of ±25 volts after alignment of the function multiplier.

A dynamic range of approximately 30 db at either input and 60 db at the output was obtained.

Overall amplitude response was flat for either or both input frequencies from d-c to 90 kc (output flat to 180 kc) with a gradual roll-off at higher frequencies.

The overall phase response at 90 kc was 65 deg and decreased almost linearly with frequency. Phase response was measured with one input a constant to make input and output frequencies identical.

Long-term drift from all causes including adjustments was within 1 percent of maximum product after an initial settling period of about 3 hours. The output zero required the longest settling time, while other adjustments reached stability more rapidly. Conventional regulated power supplies fed by a 2-percent a-c line regulator were used to power the multiplier during the stability measurements.

Multiplier Adjustment

Circuits employed to instrument the basic multiplier equation produce a nominal over-all scale factor other than one-to-one. No effort has been made to achieve a unity scale factor. The circuits, unless compensated, may introduce a number of extraneous terms that arise from misalignments.

Magnitudes of errors produced by some of the potential sources of extraneous terms, such as deviations of the effective gains of the inverters from unity or the sum-and-difference network from equality, depend upon the accuracy and stability of passive resistive components. Errors contributed by these parts of the multiplier may be minimized during construction by use of accurate and stable resistors and are therefore not considered.

Other errors, more subject to variation with time (those dependent upon the stability of active or replaceable components) are best eliminated by adjustments. Equation 1 may be rewritten to include these terms and their adjustments, in the form

$$Kxy + \Delta = A_1(x + y + z_1)^2 - A_2(x - y + z_2)^2 + C \quad (3)$$

where, in the above equation factor K is the over-all scale factor of the multiplier and Δ is the total error at the output caused by misalignments.

Scale factors of the squared sum-and-difference channels, A_1 and A_2, include the square-law, cathode-follower and subtractor stages.

The off-center terms at the inputs to the square-law stages, z_1 and z_2 include the respective square-law stage centering voltages, e_0, and the inverter zero adjustments, B, used to set them to zero

$$z_1 = (z_0)_1 + B_1 + B_2 \quad (3A)$$

$$z_2 = (e_0)_2 + B_1 - B_2 \quad (3B)$$

The term C is the (zero signal) off-zero term at the output of the multiplier. It includes the subtractor zero and also any d-c unbalance present in the cathode followers or between the outputs of the square-law stages.

Equation 3 may be expanded into the desired product and three types of error terms by carrying out the indicated operations

$$Kxy + \Delta = 2(A_1 + A_2)xy$$

$$\text{(A)} + (A_1 - A_2)(x^2 + y^2)$$

$$\text{(B)} + 2(A_1z_1 - A_2z_2)x + 2(A_1z_1 + A_2z_2)y$$

$$\text{(C)} + A_1z_1^2 - A_2z_2^2 + C$$

square-law error (A)
linear error (B)
constant error (C)
} error terms

A procedure has been devised for systematically eliminating these error terms in a convergent manner. Some method of observing the form of the multiplier output as a function of an input signal is required. Error terms are then eliminated in the order shown in Eq. 4 by the following adjustment routine:

Equating A_1 and A_2 cancels the square-law error, Eq. 4A. This is done by setting y to zero and observing the form of the output as a function of x as A is varied. When the plot is a straight line A_1 and A_2 are equal.

The next two steps eliminate the

linear error terms, Eq. 4B, by setting B_1 and B_2 so that $z_1 = z_2 = 0$. The conditions for this are found from simultaneous solution of Eq. 3A and 3B to be

$$B_1 = -(e_0)_1 + (e_0)_2/2 \quad (5A)$$

and

$$B_2 = -(e_0)_1 - (e_0)_2/2 \quad (5B)$$

The order in which B_1 and B_2 are adjusted is of no consequence.

The term B_1 is adjusted to the criterion of Eq. 5A by setting y to zero and observing the form of the output as a function of x as B_1 is varied. The observed output, Δ, will be a constant for the proper adjustment of B_1.

The term B_2 is adjusted in a similar manner by setting x to zero and observing the form of the output, Δ, as a function of y, while varying B_2. The output will again be a constant for the proper adjustment of B_2 to the criterion of Eq. 5B.

The final step is to adjust C so that the multiplier output is at zero potential with respect to ground when the input terms are zero.

Performance Limitations

Accuracy of the present multiplier stays within close limits for input signals up to a certain level and gradually deteriorates as the inputs get larger in a manner similar to gradual overloading. The error appears to be produced principally by variable current flow between deflection plates and cathode of the square-law tubes, that occurs when the deflection plates exceed a certain positive potential with respect to the cathode. The IR drops produced in the sum-and-difference network by these changes in current are equivalent to shifts of the centering voltage adjustments from their original settings and introduce corresponding errors.

The ideal remedy is to modify the tube design to minimize the deflection currents. It is possible that some of the other developmental square-law tubes already built possess improved characteristics in this respect. Short of this, more accurate multiplier performance is attainable with the same square-law tubes by finding operating conditions with lower deflection currents. Reducing the impedance levels at the inputs to the square-law stages (sum-and-difference network) also improves performance. Inverters with higher output power ratings would then be required to maintain the original amount of drive.

Frequency response is at present limited by the bandwidths of the plug-in amplifiers. The QK-329 square-law tubes have been successfully operated from d-c to vhf. They contributed no measurable phase shift to the over-all multiplier phase shift mentioned in the previous section on performance. Increases in multiplier frequency response of 100-to-1 over the present model could thus be made before the square-law tubes offered a direct obstacle to such improvements. For efficient design, a multiplier with two identical inputs should have a differential amplifier at its output with twice the bandwidth of the input.

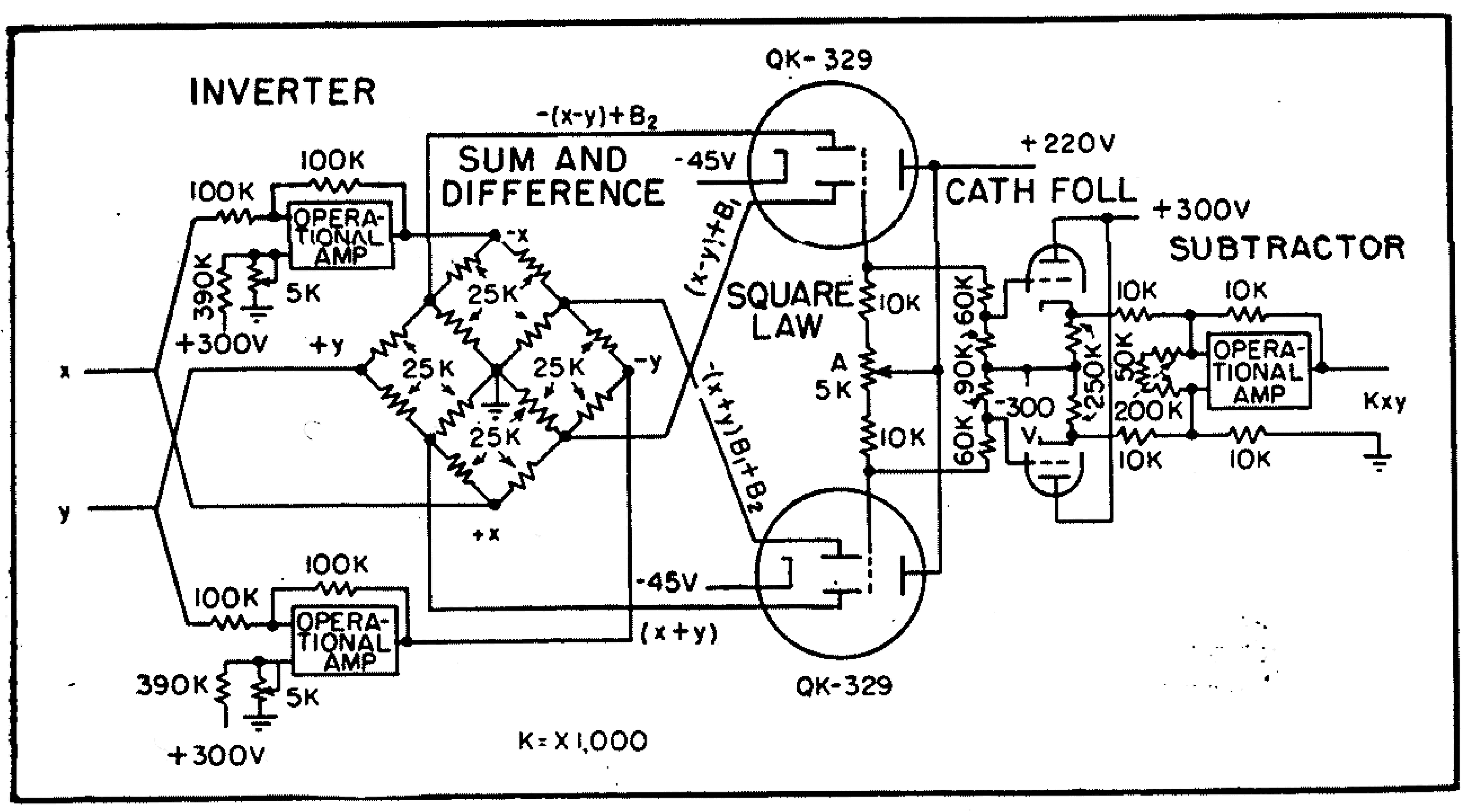


FIG. 3—Schematic of complete multiplier whose performance is primarily limited by circuitry rather than the square-law tubes

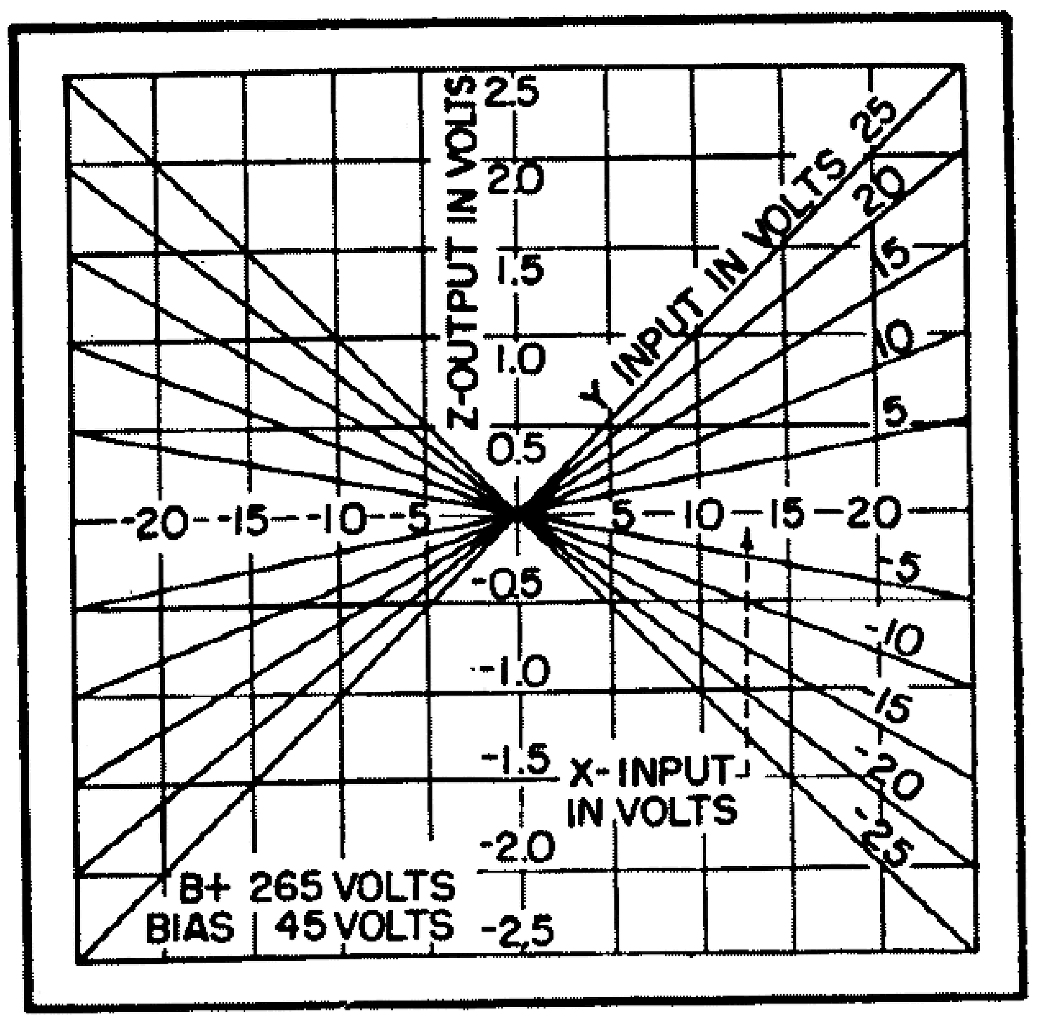


FIG. 4—Plot of multiplier characteristics, Z-KXY, using square-law tubes

Stability

Multiplier drift is primarily zero drift of the output circuits. If the maximum-output signal level were increased to make full use of the capabilities of the differential amplifier, the percent drift would be improved. Use of drift-stabilized amplifiers would also help.

The square-law stages of the present multiplier contribute only 12 percent of the observed overall long-term drift at the multiplier output or approximately 0.12 percent of maximum output. The differential output of the multiplier circuit provides a degree of inherent discrimination against the effects of drift that could result from changes in total current in the square-law tubes.

Sizeable increases in bandwidth, accuracy and zero stability are achievable with existing square-law tubes by improving the associated circuitry. Square-law tubes of the general type employed, therefore, provide a nucleus around which considerable forward progress in analog multipliers can be made.

The work described here was supported in part by the Army Signal Corps, the Navy Dept. (O.N.R.) and the Air Force (A.M.C.) under contracts DA-36-039sc100 and 3-99-10-022 with the Research Laboratory of Electronics, M.I.T.

The authors wish to acknowledge the contributions of J. Gradijan, G. Fine and A. Moccia of the Air Force Cambridge Research Center.

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